JFM NOTEBOOKS

In Search of Stratified Turbulence

JFM **983** A20 (2024)

Sam Lewin
(et al.)

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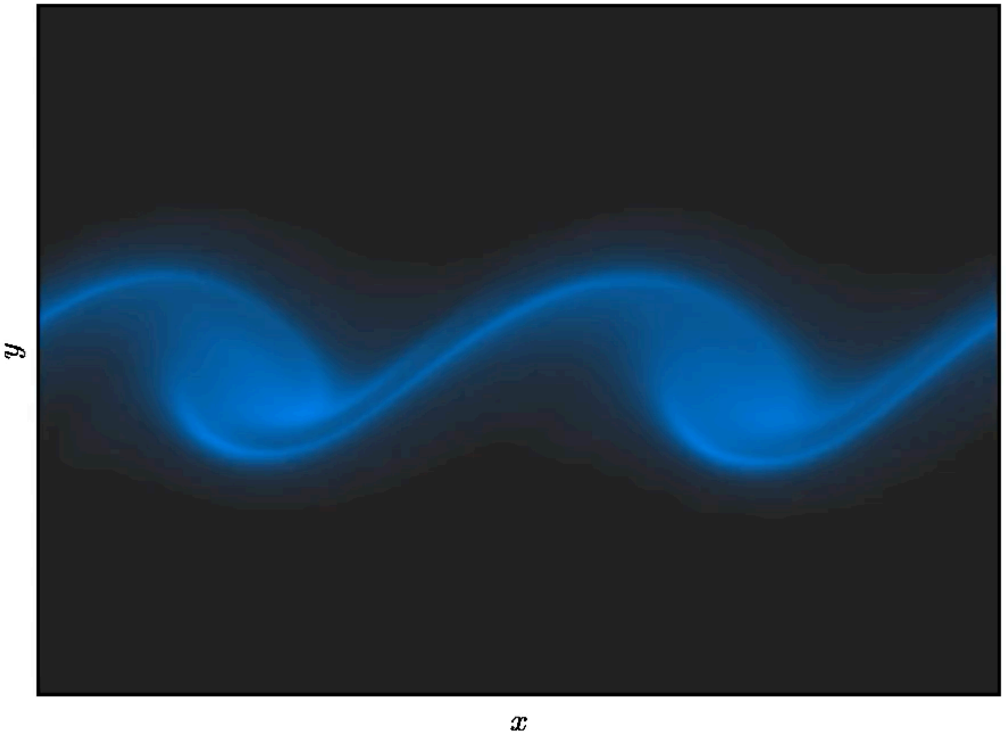
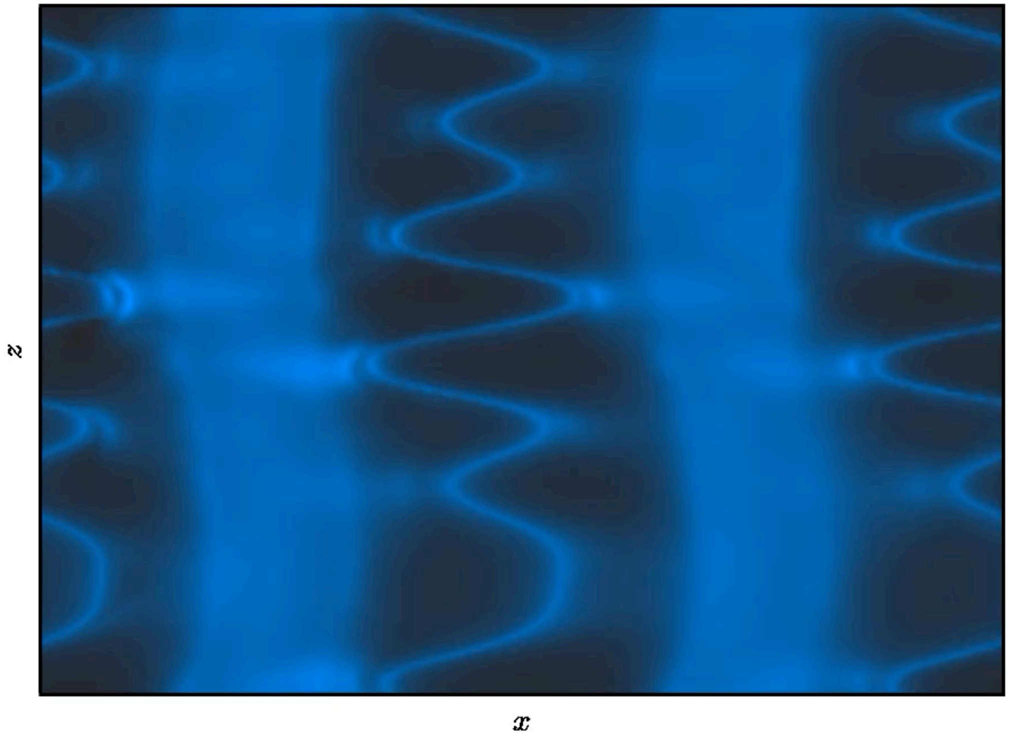
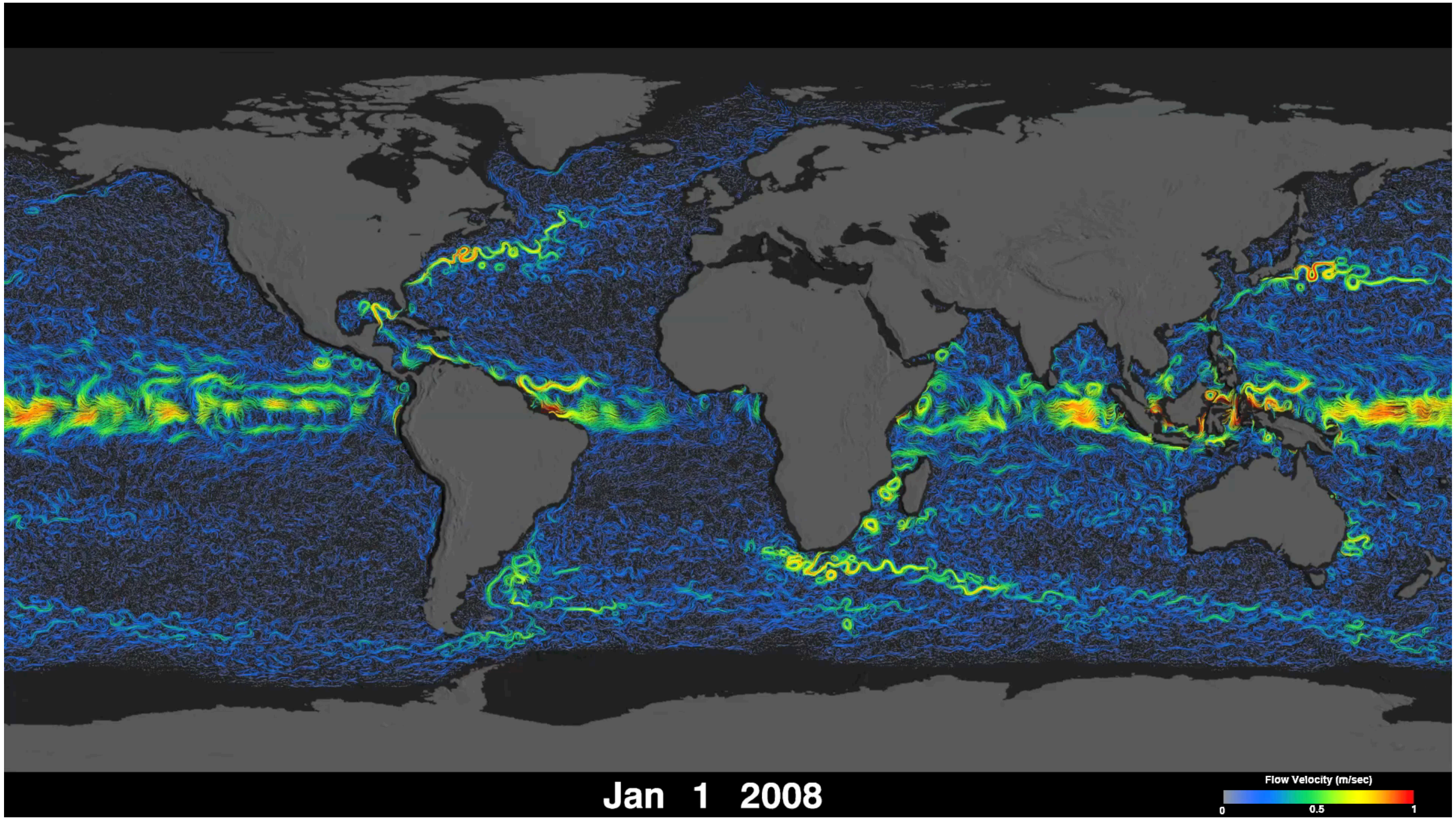




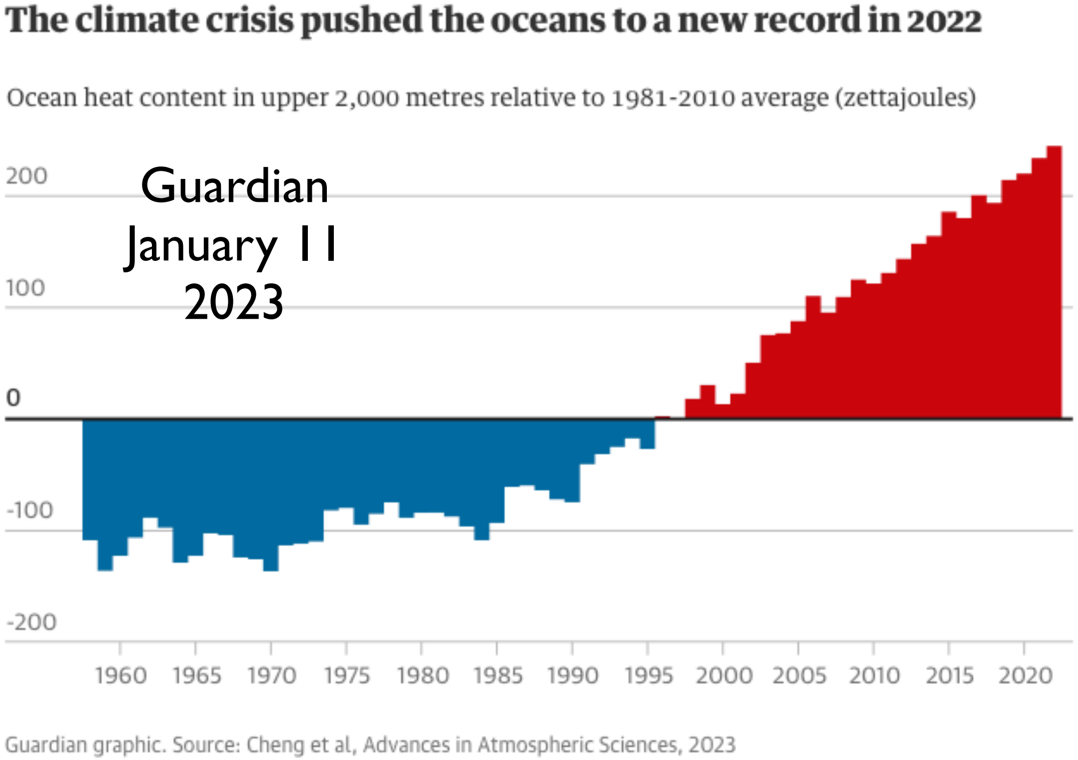
Stratified Turbulence: Who cares?

- Ocean dynamics key to climate emergency: global weather and climate controlled by oceans

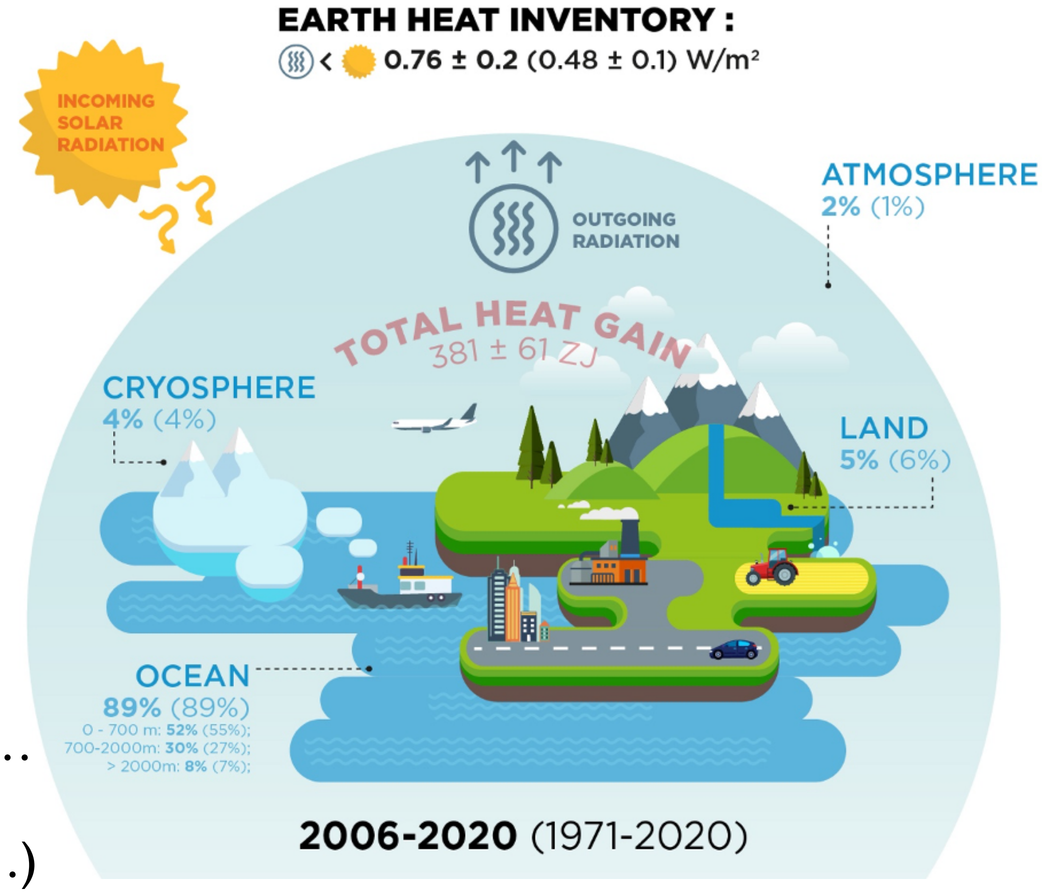


OSCAR (Dohan: Earth & Space Research)

Stratified Turbulence: Who Cares?



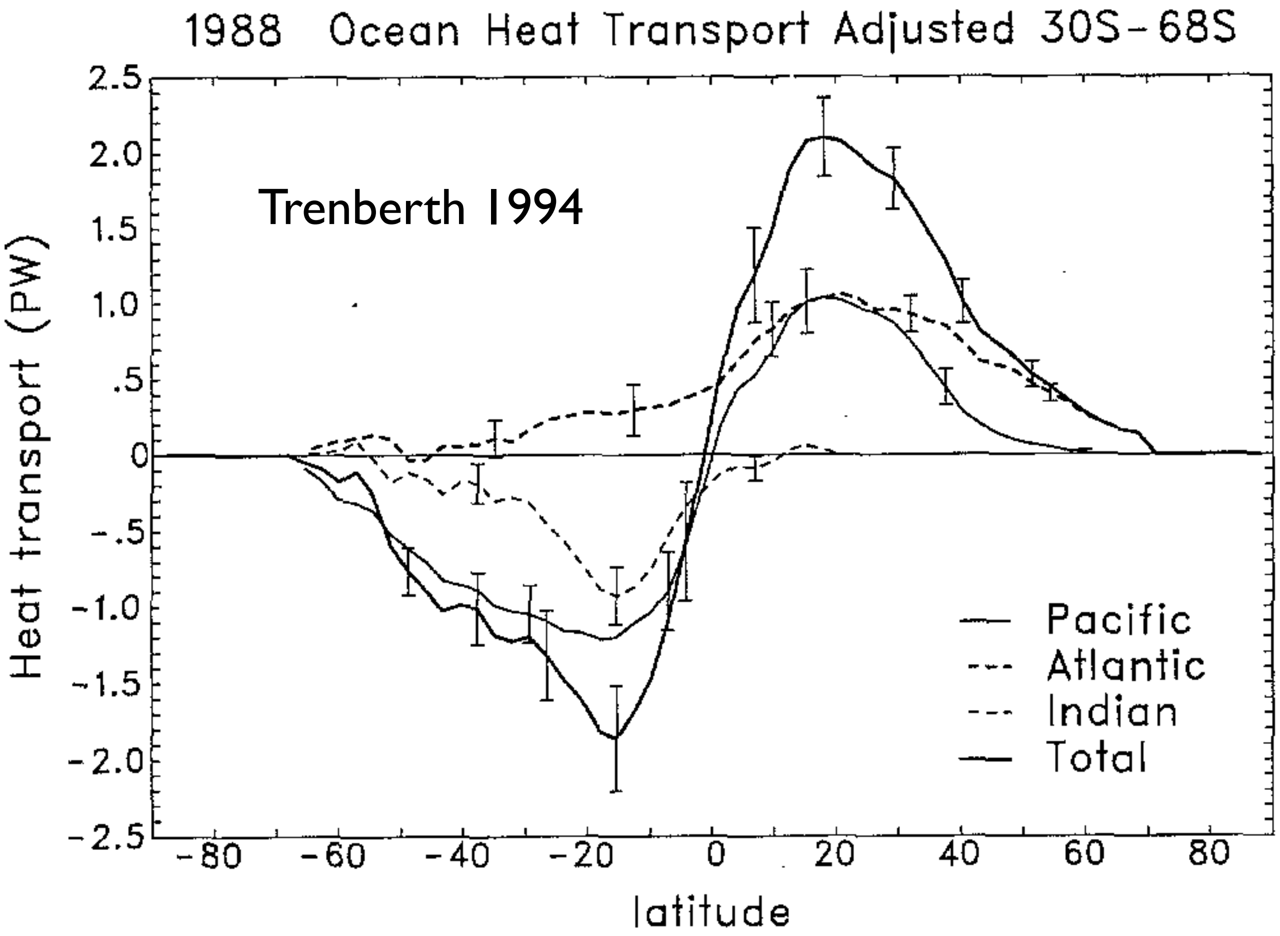
Guardian
January 11
2023

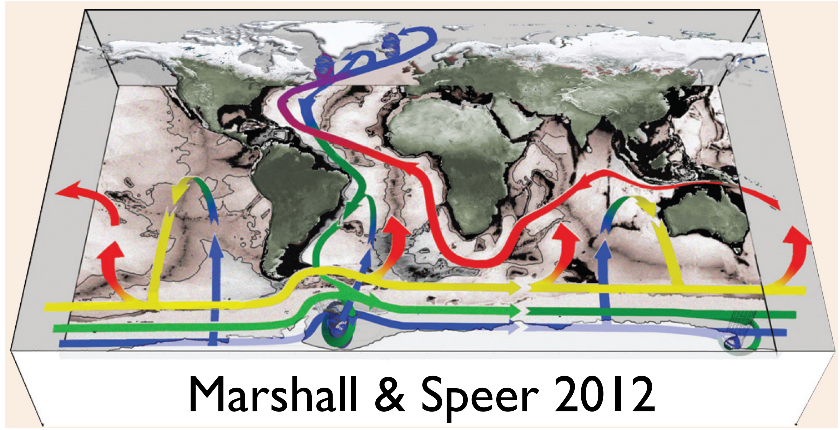


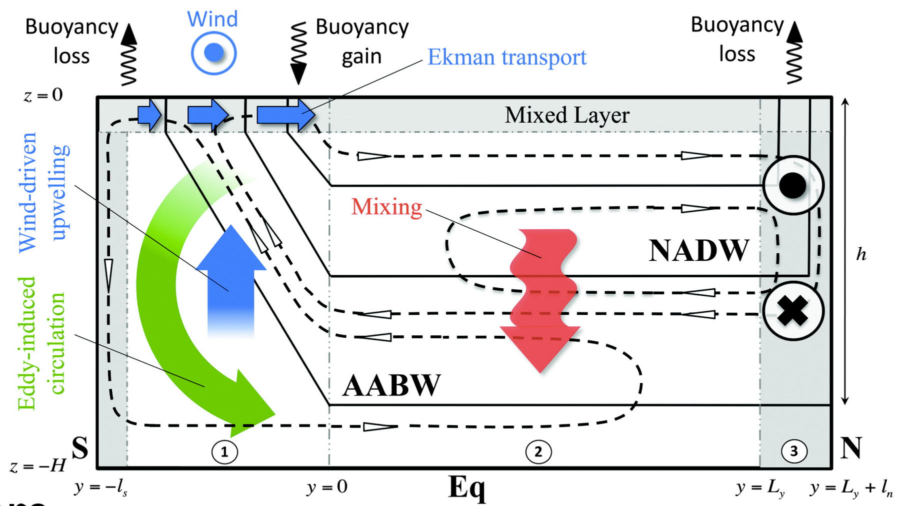
Past results no guarantee of future performance…
Models can't just be "tuned"
90% of excess heat is in oceans (at the moment…)
3.4m of Ocean has same heat capacity as 100km of Atmosphere
How does small-scale mixing lead to observed stratification?

Schuckmann et al. 2023

Stratified Turbulence: Who Cares?



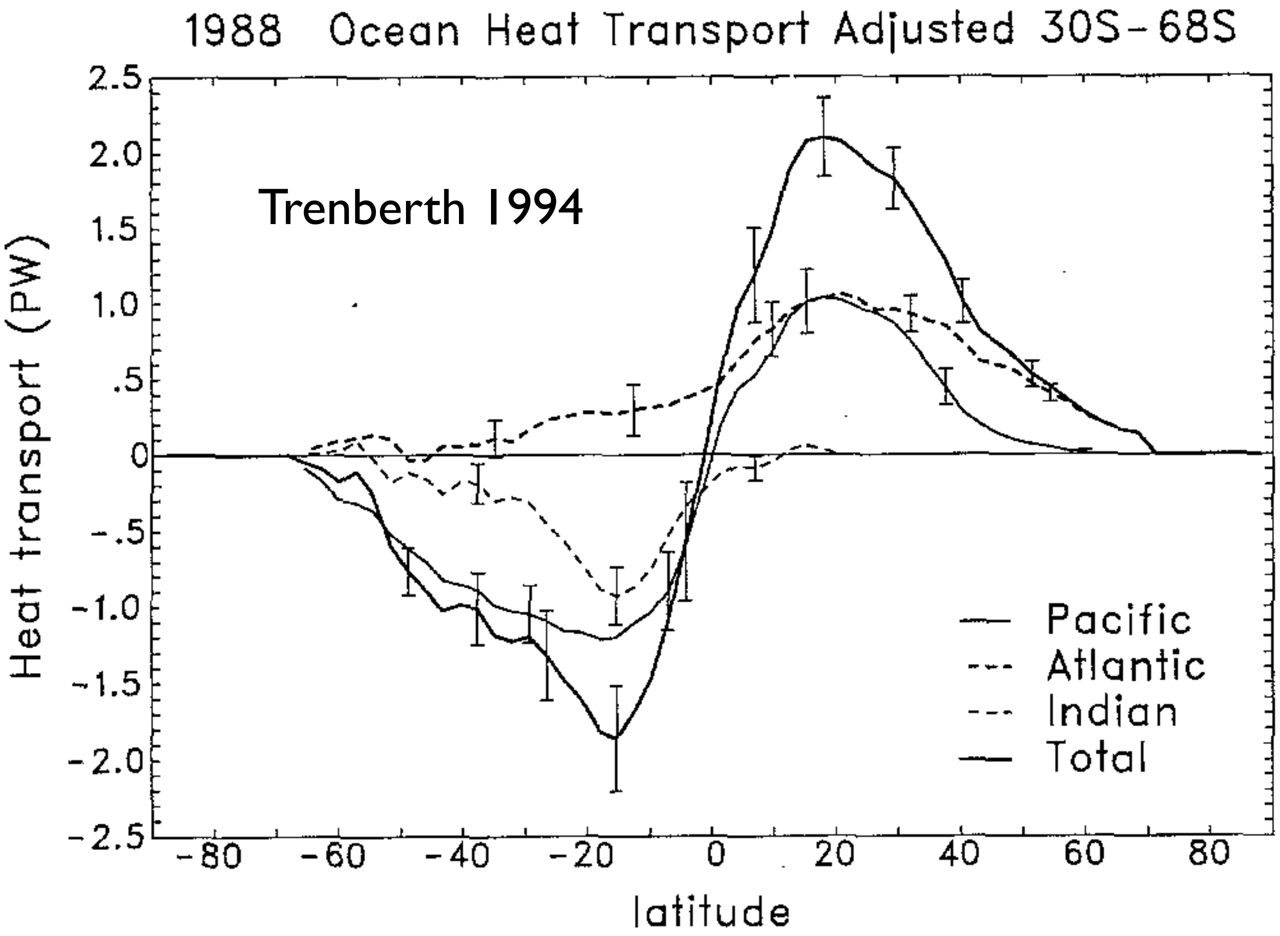


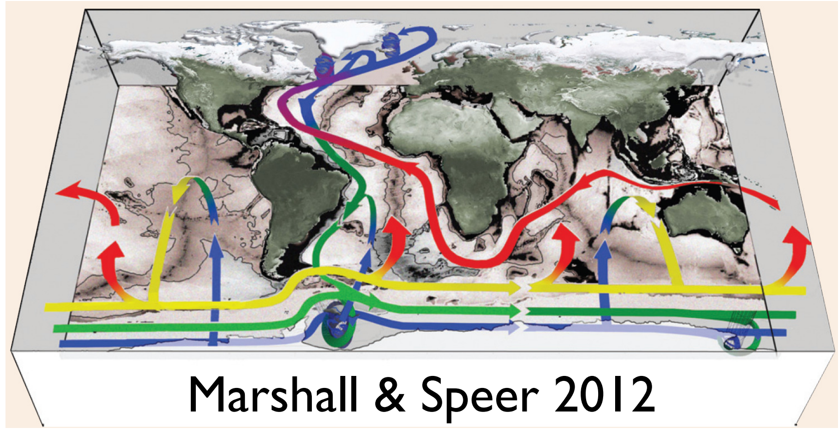


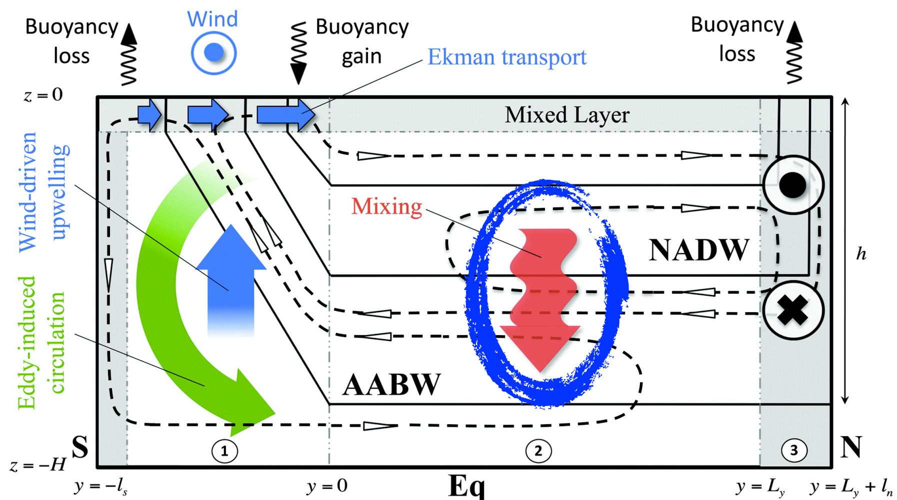
Nikurashin & Vallis 2012

3.4m of ocean has same heat capacity as 100km of atmosphere
Mixing of density/heat key component for closing circulation

Stratified Turbulence: Who Cares?





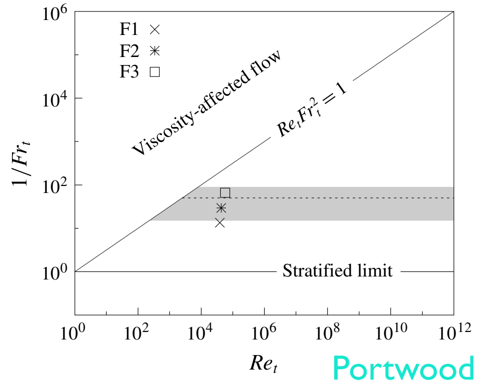


3.4m of ocean has same heat capacity as 100km of atmosphere
Mixing of density/heat key component for closing circulation

Nikurashin & Vallis 2012

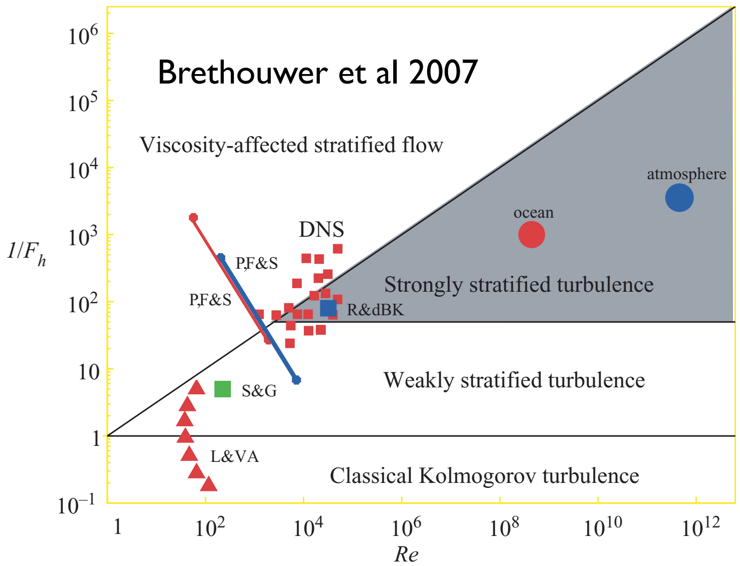
Layered Anisotropic Stratified Turbulence

- Apparently atmosphere/ocean: $Re_H \equiv \dfrac{U_H L_H}{\nu} \gg 1; \; Fr_H \equiv \dfrac{U_H}{N L_H} \ll 1$
- Challenging to access numerically: how triggered or forced?



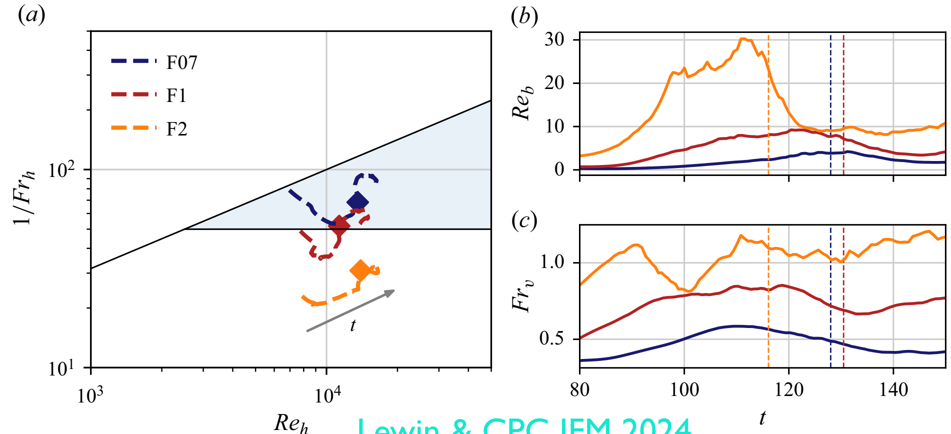
Portwood et al JFM 2016

FIGURE 1. Regime diagram in terms of Re_t and Fr_t following Brethouwer *et al.* (2007). The grey band represents the range of estimates for the lowest value of $1/Fr_t$ in the LAST regime based on the range for A reported in the literature. The dashed line indicates the limit of the LAST regime assuming $A = 1$. The three symbols mark the parameter values for the simulations discussed in detail in this paper.



$$N^2 = -\frac{g}{\rho_0}\frac{\partial \overline{\rho}}{\partial z}$$

FIGURE 18. Regimes in stably stratified flows. The conditions under which our and other DNS and experiments are carried out are represented by symbols. Red squares (labelled DNS): present DNS; blue square (R&dBK): DNS run F4R64 by Riley & deBruynKops (2003); green square (S&G): DNS run A by Staquet & Godeferd (1998); red triangles (L&VA): experiment of decaying stratified turbulence by Lienhard & Van Atta (1990) listed in their table 1; red and blue lines (P,F&S): experiments bc and be of decaying stratified turbulence by Praud *et al.* (2005). Conditions typically found in the middle atmosphere (Lindborg 2006) and the upper ocean (Moum 1996) are shown by the blue and red circle respectively, but these conditions can vary considerably. Values of Re and F_h are estimated using (2.12).

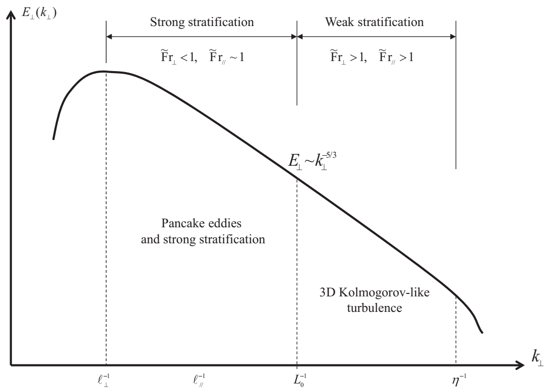


Lewin & CPC JFM 2024

Figure 7. (*a*) Trajectories of Re_h vs $1/Fr_h$ for simulations F07, F1 and F2. The direction of time is from left to right, indicated by the grey arrow. Markers indicate the points at which the 'fully turbulent' snapshots considered in figures 8 and 9 are taken. The light blue shaded region denotes the 'strongly stratified' region of parameter space delineated by Brethouwer *et al.* (2007) according to $Re_h Fr_h^2 > 1$ and $Fr_h < 0.02$. Panels (*b*,*c*) show the evolution of the buoyancy Reynolds number Re_b and vertical Froude number Fr_v for each simulation, where the dashed lines correspond to the time instant at which the markers are located in (*a*). The directory including the data and Jupyter notebook for producing the figure can be found at https://cocalc.com/Cambridge/S0022112024001216/JFM-Notebooks/files/fig7.

Layered Anisotropic Stratified Turbulence?

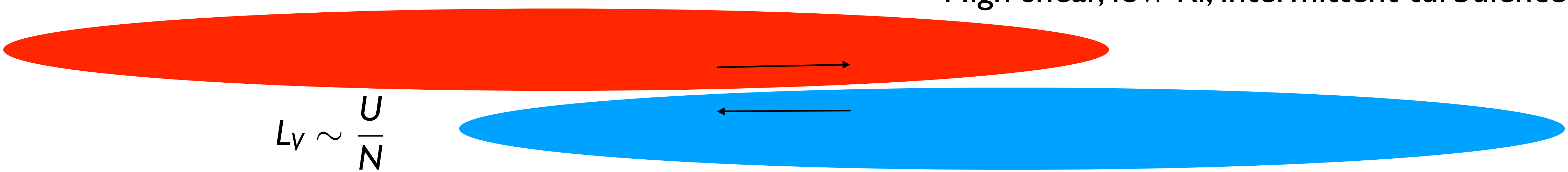
- Large buoyancy Reynolds number ensures wide separation between Ozmidov & Kolmogorov scales:
- Gives some chance of isotropic inertial range $Re_b \equiv \frac{\epsilon}{\nu N^2} = \left[\left(\frac{\mathcal{E}}{N^3} \right)^{1/2} \left(\frac{\mathcal{E}}{\nu^3} \right)^{1/4} \right]^{4/3} \equiv \left(\frac{L_O}{L_K} \right)^{4/3}$
- Particularly if Ozmidov scale is ALSO forcing injection scale
- Scaling arguments: vertical buoyancy scale $L_V \sim \frac{U}{N}$ separate from L_O
- Layered Anisotropic Stratified Turbulence: Strongly Stratified Turbulence?
- Arguments of Billant/Chomaz/LindborgBrethouwer: $L_H \gg L_V \gg L_O \gg L_K$



Davidson 2013

$$Re_H \equiv \frac{U_H L_H}{\nu} \gg 1; \;\; Fr_H \equiv \frac{U_H}{N L_H} \ll 1; \;\; Fr_V \equiv \frac{U_H}{N L_V} \sim 1; \;\; \mathcal{E} \sim \frac{U^3}{L_H} \rightarrow Re_H Fr_H^2 \gg 1 \leftrightarrow Re_b \gg 1$$

High shear, low Ri, intermittent turbulence



$$L_V \sim \frac{U}{N}$$

LAST from an Initial Value Problem?

- How might this LAST regime appear in a freely evolving flow from some ICs without forcing?
- Vertical overturning strongly suppressed by strong stratification: classic instabilities useless…(M-H)
- Wall-bounded flows: inevitably "weak" (Zhou et al 2017) body-forced flows inherently artificial…
- Horizontal instabilities with vertical vorticity coupled to non-normal lift-up & zig-zag leads to LAST!

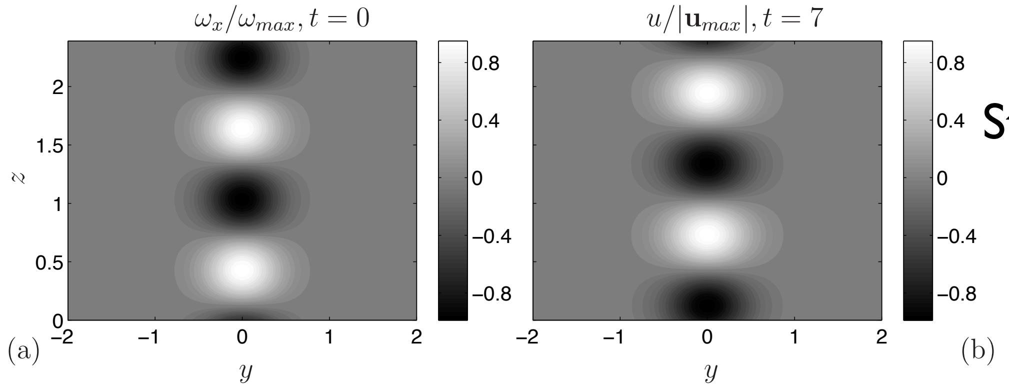
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad \bar{v}=\int_{-\infty}^{\infty} v\,dx \quad \nabla^2 p=-2U'\frac{\partial v}{\partial x}+\frac{1}{Fr^2}\frac{\partial \hat{\rho}}{\partial z}\rightarrow \nabla^2\bar{p}=0\rightarrow \bar{p}=0$$

$$\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}+vU'=-\frac{\partial p}{\partial x}+\frac{1}{Re}\nabla^2 u \quad \frac{\partial \bar{v}}{\partial t}\simeq-\frac{\partial \bar{p}}{\partial y}\rightarrow \bar{v}\simeq\bar{v}_0\rightarrow \bar{u}\simeq\bar{u}_0-\bar{v}_0U't$$

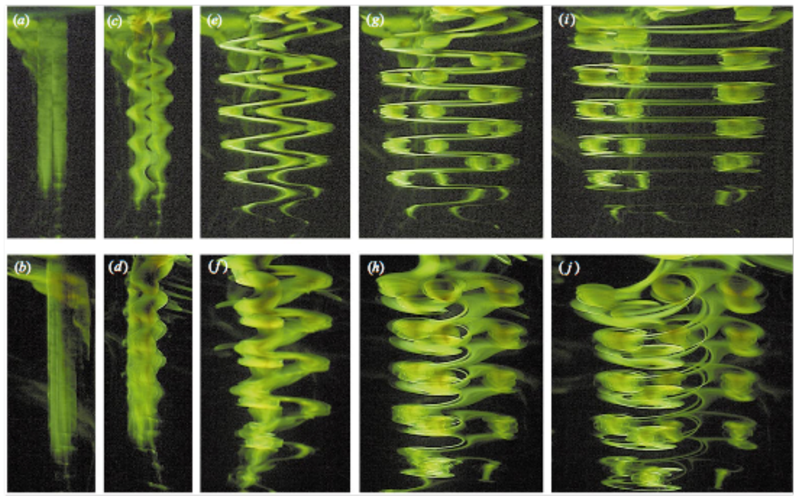
$$\frac{\partial v}{\partial t}+U\frac{\partial v}{\partial x}=-\frac{\partial p}{\partial y}+\frac{1}{Re}\nabla^2 v$$

$$\frac{\partial w}{\partial t}+U\frac{\partial w}{\partial x}=-\frac{\partial p}{\partial z}+\frac{1}{Re}\nabla^2 w-\frac{1}{Fr^2}\hat{\rho}$$

Ellingsen & Palm 1975/Landahl 1980
(Hoiland/Moffatt 1967…)



Nonmodal
Streamwise Vorticity
Arratia 2011



Zig-zag Instability of Vertical Vortices
Naturally forms layers ~ U/N
Billant & Chomaz 2000

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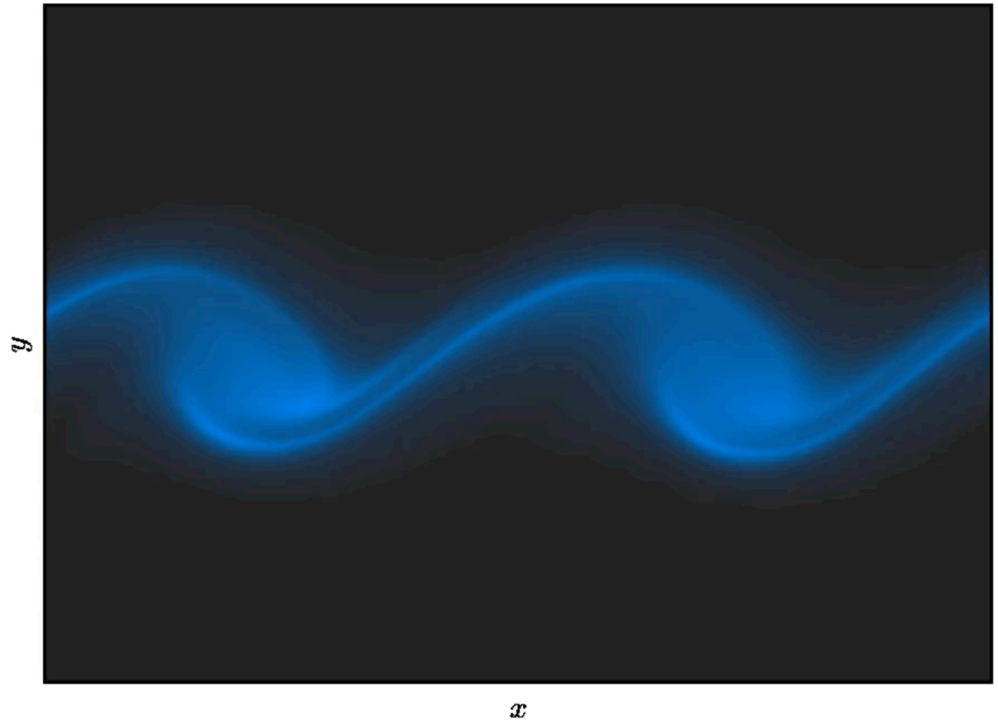
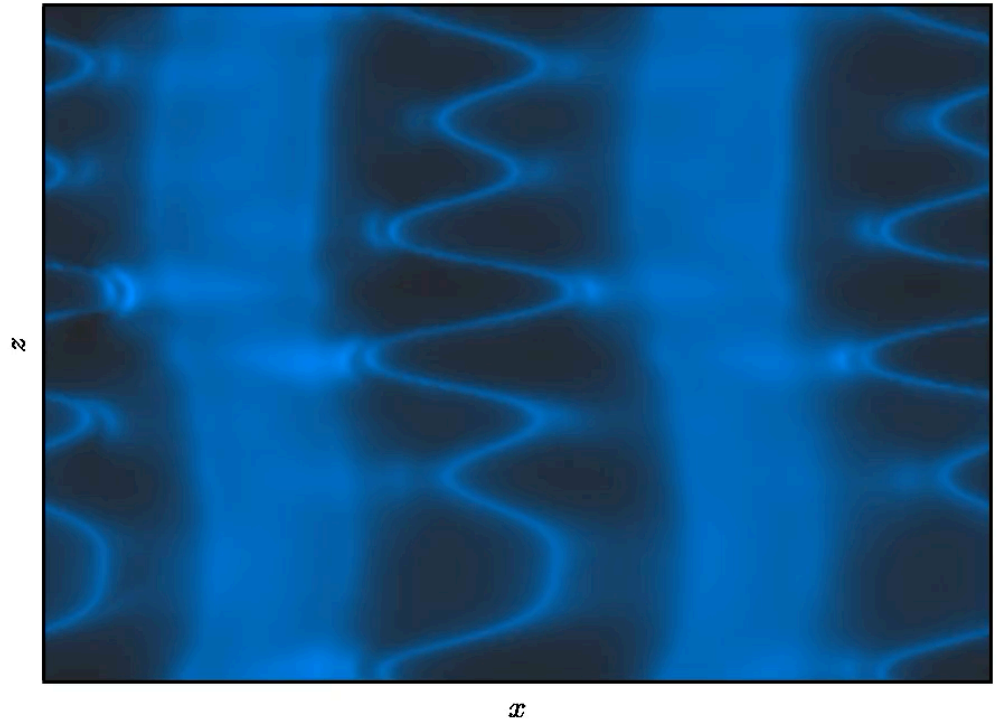
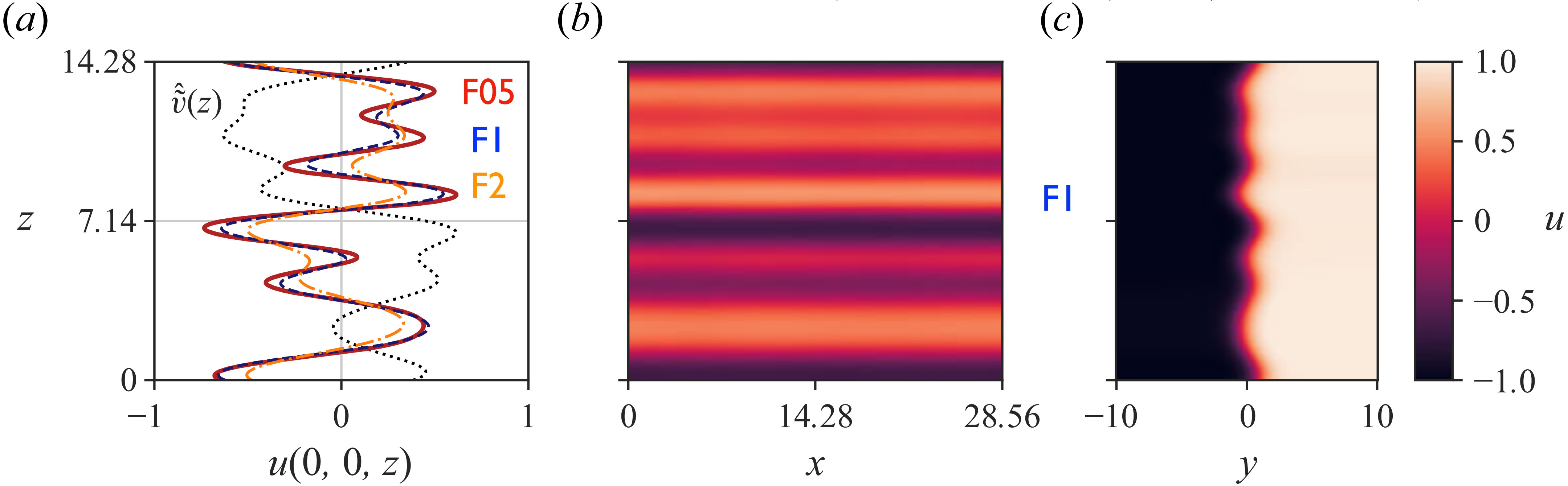




Lewin & CPC JFM 2024 (with notebooks)

LAST from an Initial Value Problem?

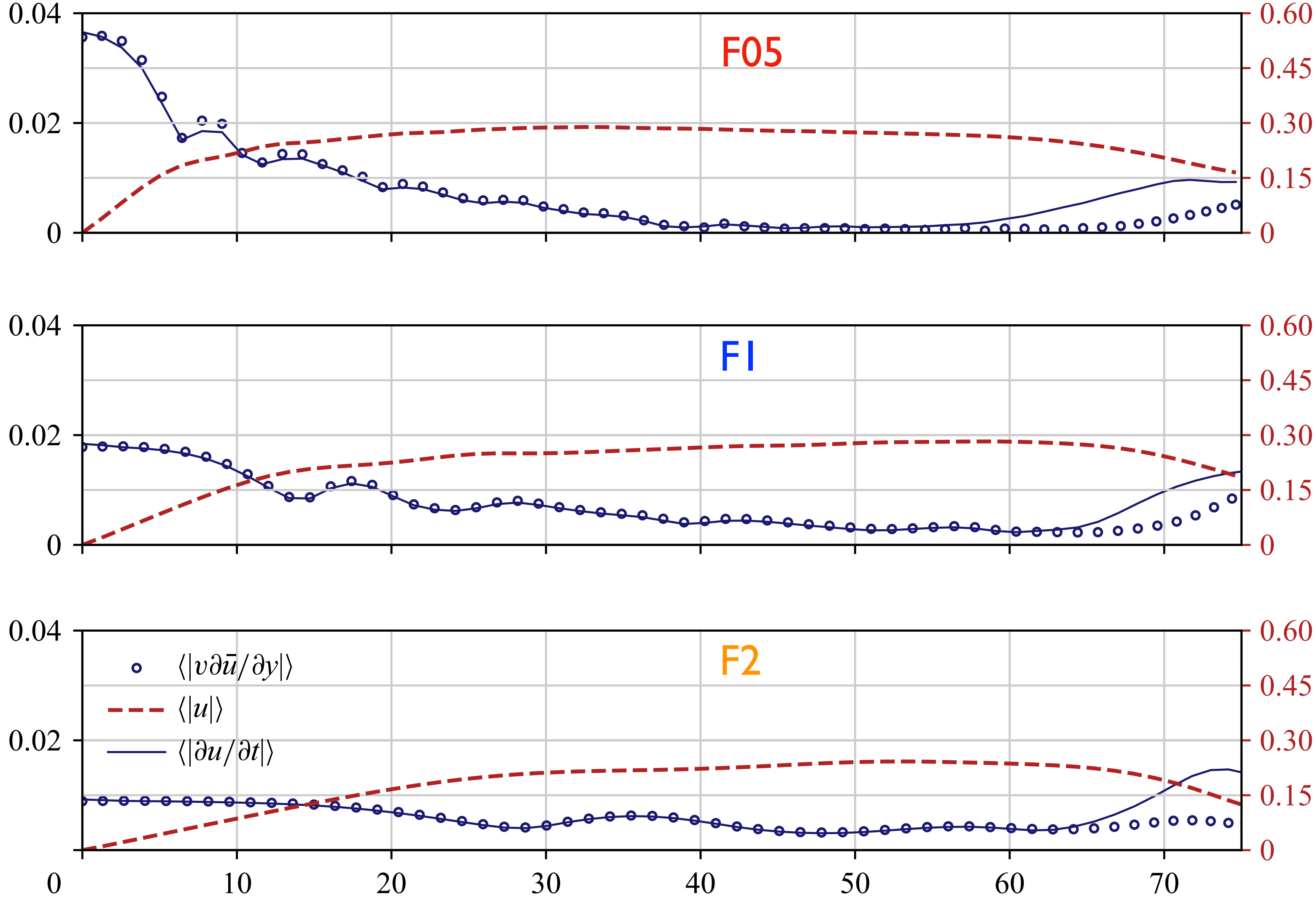
Simulation	Fr_0	Ri_0	Re_0	(L_x, L_y, L_z)	(N_x, N_y, N_z)
F05	0.5	4	2000	(28.56, 66.34, 14.28)	(512, 673, 256)
F07	0.71	2	2000	(28.56, 66.34, 14.28)	(1024, 896, 512)
F1	1	1	2000	(28.56, 66.34, 14.28)	(1024, 896, 512)
F2	2	0.25	2000	(28.56, 66.34, 14.28)	(1024, 896, 512)



- Perturbing v(z) leads to layering in u at a different vertical scale: is it “lift-up”?

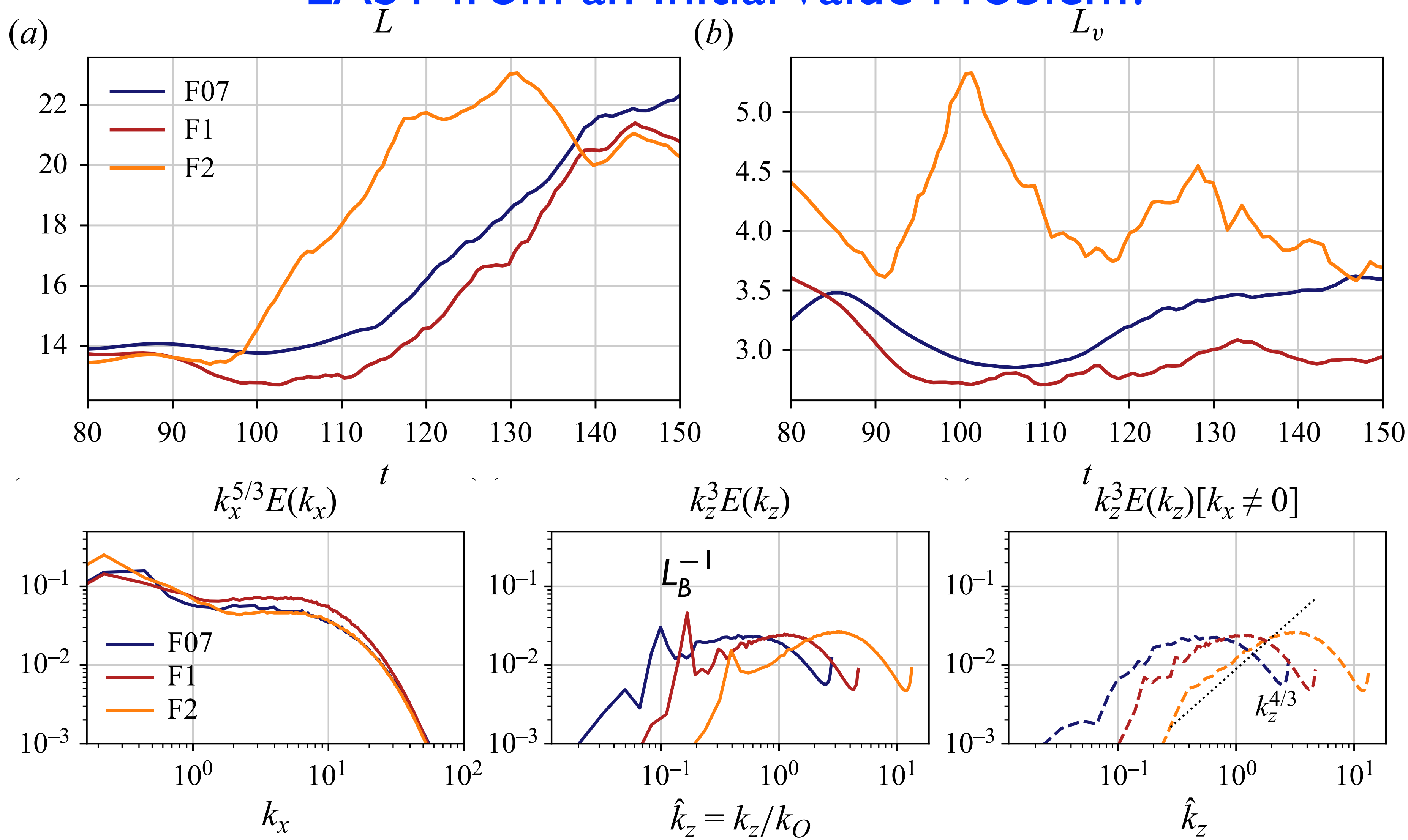
Lewin & CPC JFM 2024 (with notebooks)

LAST from an Initial Value Problem?



- Good agreement of $\partial u/\partial t \simeq vU'$ and u increases approximately linearly…but is it LAST?

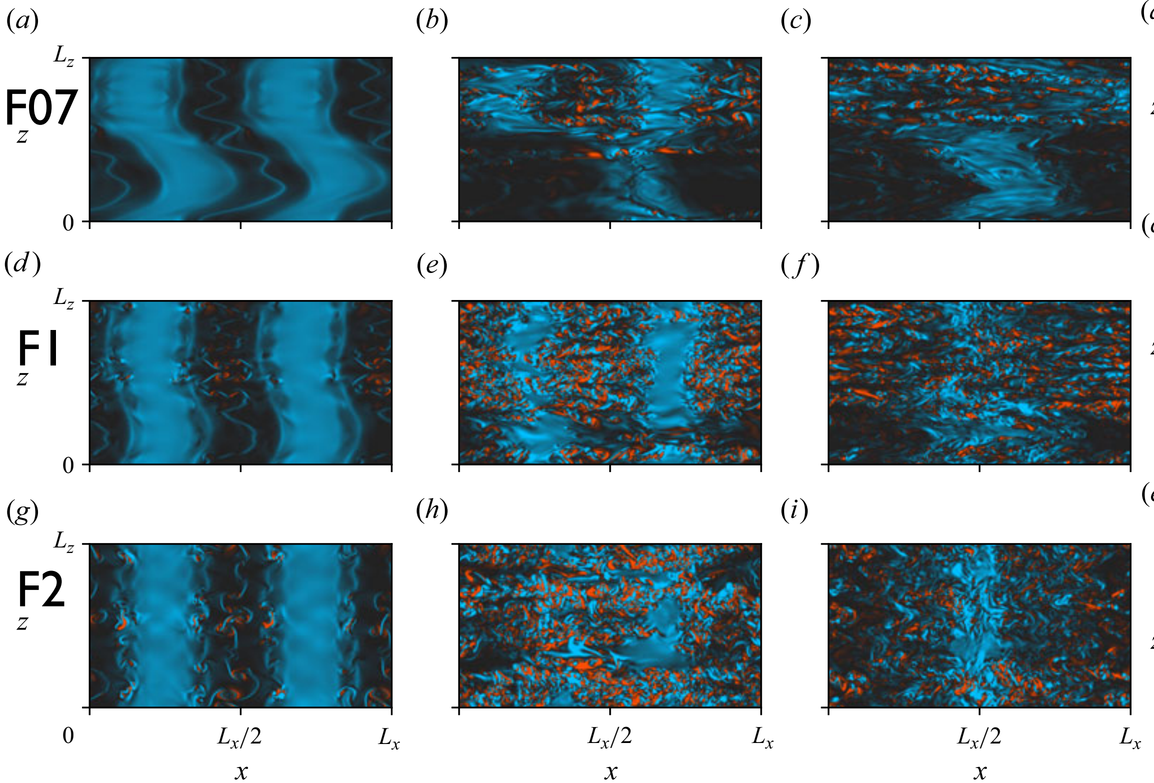
LAST from an Initial Value Problem?

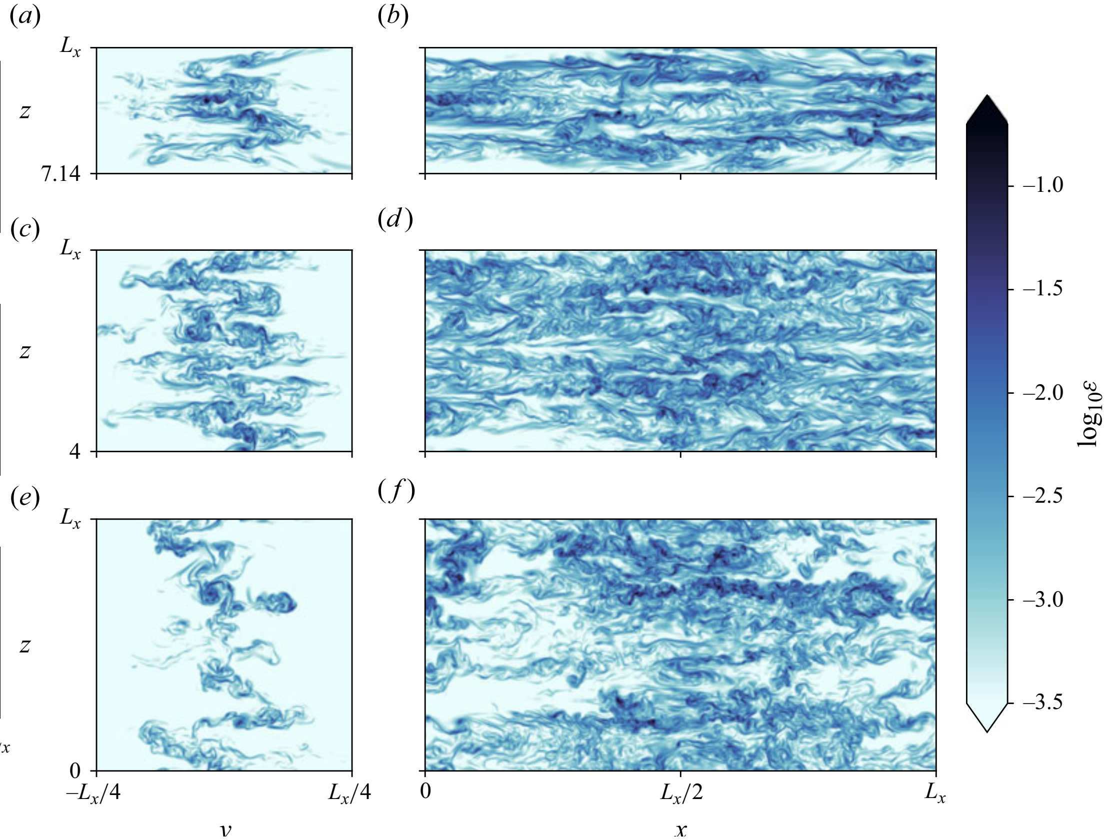


- Horizontal scales grow; vertical scales saturate; spectra are plausible…mechanisms?

LAST from an Initial Value Problem?

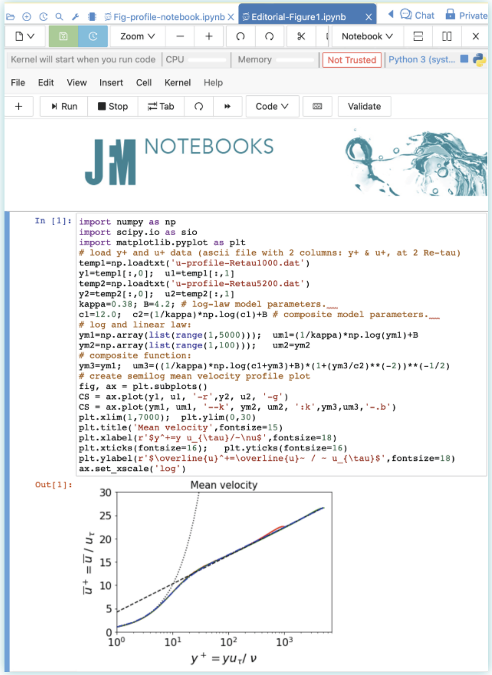
- Building on work from Basak & Sarkar JFM 2006: zig-zag-like dynamics seem to occur
- Lift-up triggers "smaller" KHI type instabilities if local Re is sufficiently high…
- Structure **and** statistics ~~almost but not quite totally unlike~~ largely consistent with LAST!





Lewin & CPC JFM 2024 (with notebooks)

Dive into the data with JFM Notebooks: replot, rescale, rotate figures in JFM



- Interactive objects that execute code and visualise data, bringing article figures out of the static, two dimensional page.
- Increase the openness, transparency and accessibility of your research, going beyond making the underlying data used to construct figures and scientific arguments available, they enable interested readers to interrogate and probe that data to explore alternative ideas and gain further insights.
- More fully convey the essence, complexity, and beauty of fluid flow phenomena.
- Improve how individual fluid mechanics researchers communicate results amongst each other to the wider community of JFM readers.

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“Conclusions”

- It’s really important to understand turbulent stratified mixing for climate modelling
- The Ocean appears to be in a strongly stratified turbulent state: how it gets there is an open question
- This state is Layered, Anisotropic, Stratified and Turbulent: not all clear how to keep it going…
- Vertically sheared flows have instabilities…only in weak stratification
- Wall-bounded flows just can’t access strongly stratified regimes
- Body-forced flows are artificial…
  - If you require (strongly) steady state: recover Osborn’s formula…and weak stratification
  - More loose forcing seems to lead to spatio-temporal intermittency: turbulence is weakly stratified
- Move away from modal instabilities to exploit transient mechanisms…but what about extremes?
- Lift-up in **horizontal** (with vertical decorrelation of perturbation) can help…is it really LAST?

LAST from an Initial Value Problem?

- Associated mixing shows (unsurprisingly) little correlation between $\mathcal{E}$ and χ …
- Mixing seems to be quite "efficient", similar to overturning rather than scouring (Woods et al 2010)
- Really transient, and not really LAST as Re_B is still too small…to say nothing of Prandtl number…

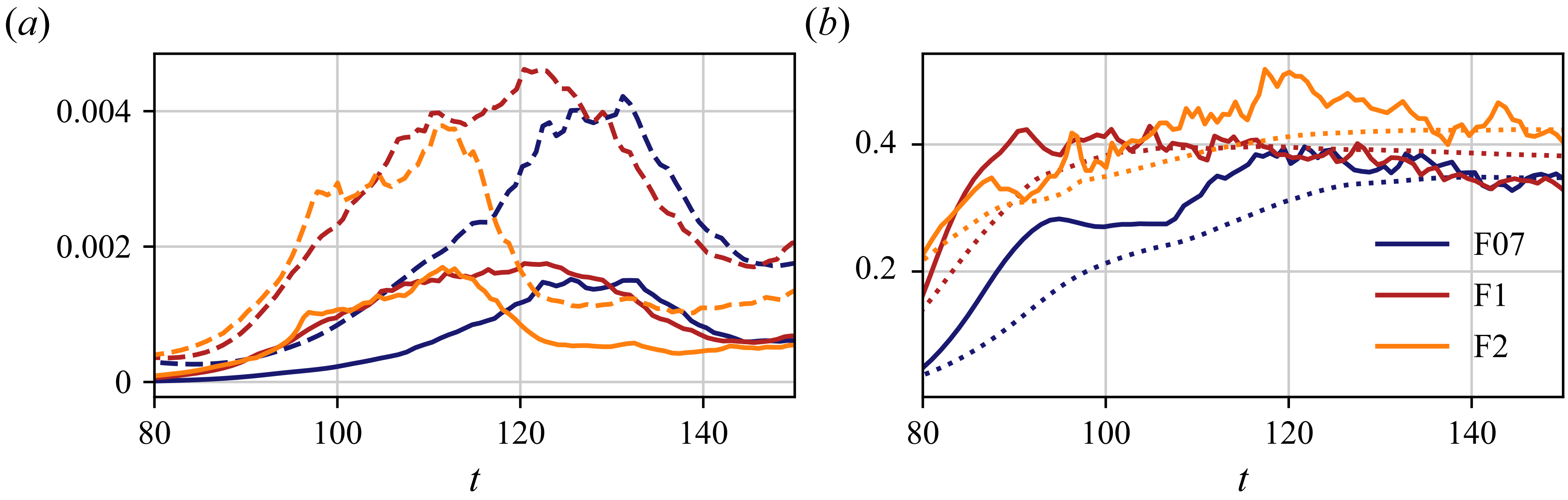
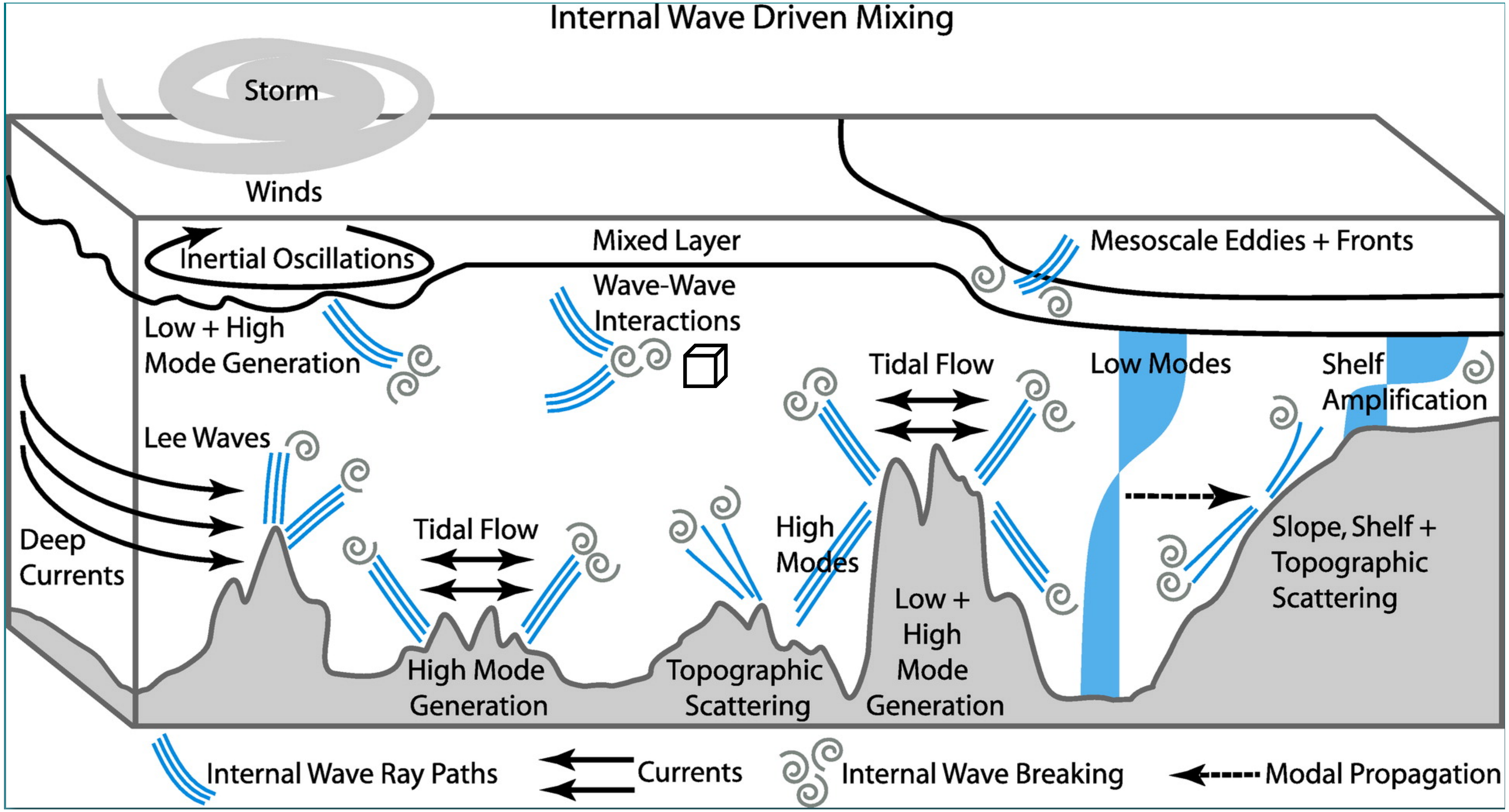


Figure 10. Time evolution of (*a*) χ (solid lines) with ε (dashed lines) superposed; (*b*) instantaneous and cumulative flux coefficients Γ_i (solid lines) and Γ_c (dotted lines). Each simulation is indicated by the colours consistent with previous figures. The directory including the data and Jupyter notebook for producing the figure can be found at https://www.cambridge.org/S0022112024001216/JFM-Notebooks/files/fig10.

Stratified Turbulence: Who Cares?

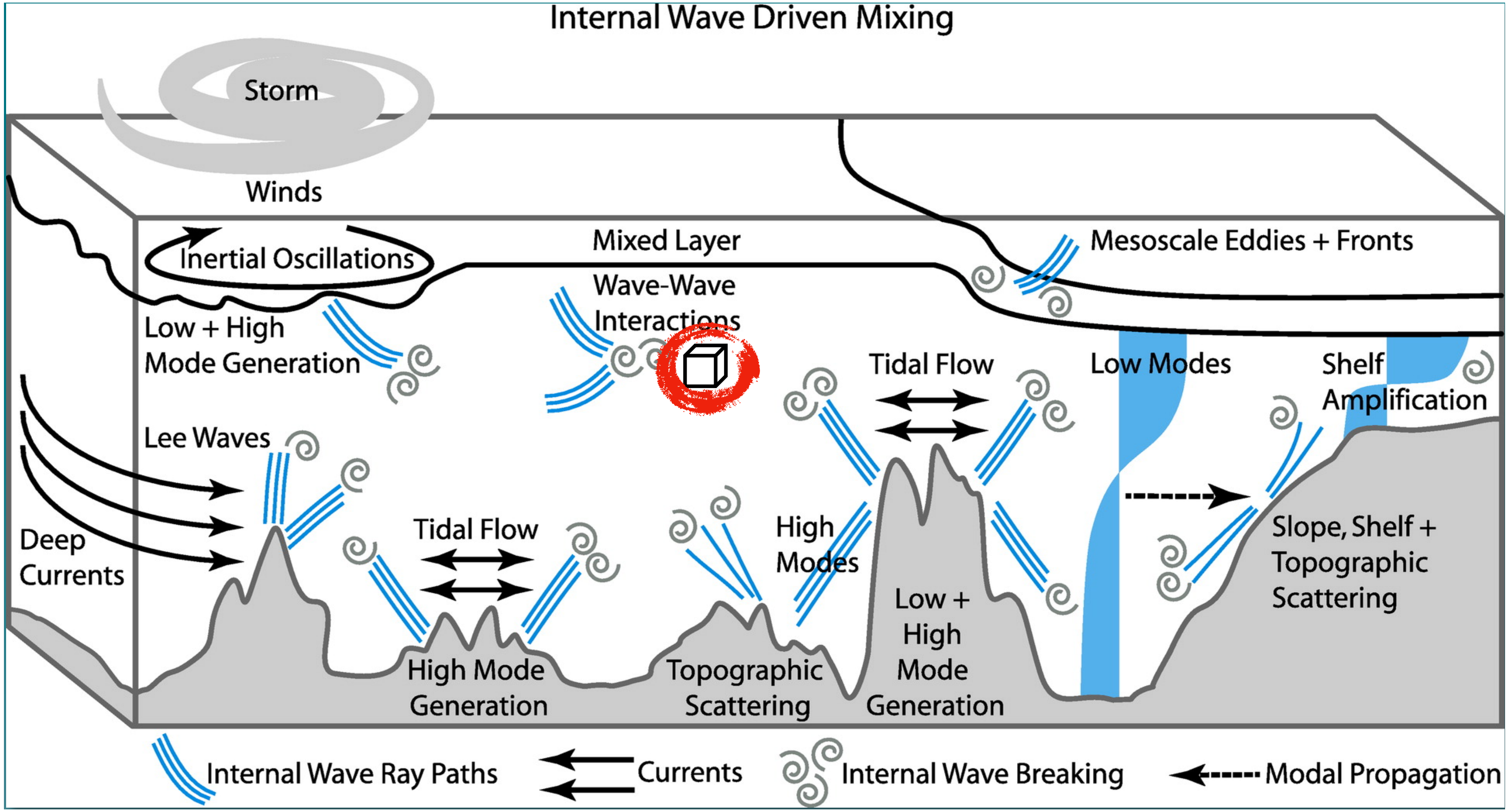
- Lots of important/complex processes:



MacKinnon et al 2017

Stratified Turbulence: Who Cares?

- Lots of important/complex processes: what can a ~~spherical cattle rustler~~ mathematician do to help?



MacKinnon et al 2017

Stratified Turbulence: Who Cares?

- Key Question: How to model vertical diffusivity of heat: $\kappa_T \equiv \dfrac{\frac{g}{\rho_0}\langle w'\rho'\rangle}{\frac{g}{\rho_0}|\partial\overline{\rho}/\partial z|} \equiv \dfrac{\mathcal{B}}{N^2}$
- Assume that it can be described as an eddy diffusivity using buoyancy frequency N:

Abyssal recipes

WALTER H. MUNK*

$$\frac{\kappa_T}{\kappa} \sim O(10^3)$$

(*Received* 31 *January* 1966)

Abstract—Vertical distributions in the interior Pacific (excluding the top and bottom kilometer) are not inconsistent with a simple model involving a constant upward vertical velocity $w \approx 1{\cdot}2$ cm day^{-1} and eddy diffusivity $\kappa \approx 1{\cdot}3$ cm^2 sec^{-1}. Thus temperature and salinity can be fitted by exponential-like solutions to $[\kappa \cdot d^2/dz^2 - w \cdot d/dz]\ T, S = 0$, with $\kappa/w \approx 1$ km the appropriate "scale height." For Carbon 14 a decay term must be included, $[\ \ ]\ {}^{14}C = \mu\ {}^{14}C$; a fitting of the solution to the observed ^{14}C distribution yields $\kappa/w^2 \approx 200$ years for the appropriate "scale time," and permits w and κ to be separately determined. Using the foregoing values, the upward flux of Radium in deep water is found to be roughly $1{\cdot}5 \times 10^{-21}$ g cm^{-2} sec^{-1}, as compared to 3×10^{-21} g cm^{-2} sec^{-1} from sedimentary measurements by GOLDBERG and KOIDE (1963). Oxygen consumption is computed at 0·004 (ml/l) year^{-1}. The vertical distributions of T, S, ^{14}C and O_2 are consistent with the corresponding south–north gradients in the deep Pacific, provided there is an average northward drift of at least a few millimetres per second.

How can one meaningfully interpret the inferred rates of upwelling and diffusion? The annual freezing of $2{\cdot}1 \times 10^{19}$ g of Antarctic pack ice is associated with bottom water formation in the ratio 43 : 1, yielding an estimated 4×10^{20} g year^{-1} of Pacific bottom water; the value $w = 1{\cdot}2$ cm day^{-1} implies 6×10^{20} g year^{-1}. I have attempted, without much success, to interpret κ from a variety of viewpoints: from mixing along the ocean boundaries, from thermodynamic and biological processes, and from internal tides. Following the work of COX and SANDSTROM (1962), it is found that surface tides are scattered by the irregular bottom into internal modes with an associated energy flux of 4×10^{-6} ergs g^{-1} sec^{-1} (one sixth the total tidal dissipation). Such internal modes can produce shear instability in the Richardson sense. It is found that internal tides provide a marginal but not impossible mechanism for turbulent diffusion in the interior oceans.

Walter Munk: GO[ceanographer]AT